## Exercise 7.1

## Question 1:

In quadrilateral $A C B D, A C=A D$ and $A B$ bisects $\angle A$ (See the given figure). Show that $\triangle A B C \cong \triangle A B D$. What can you say about $B C$ and $B D$ ?


Answer:
In $\triangle A B C$ and $\triangle A B D$,
$A C=A D$ (Given)
$\angle C A B=\angle D A B$ (AB bisects $\angle A$ )
$A B=A B$ (Common)
$\therefore \triangle A B C \cong \triangle A B D$ (By SAS congruence rule)
$\therefore B C=B D(B y C P C T)$
Therefore, $B C$ and $B D$ are of equal lengths.
Question 2:
$A B C D$ is a quadrilateral in which $A D=B C$ and $\angle D A B=\angle C B A$ (See the given figure).
Prove that
(i) $\triangle A B D \cong \triangle B A C$
(ii) $\mathrm{BD}=\mathrm{AC}$
(iii) $\angle \mathrm{ABD}=\angle \mathrm{BAC}$.


Answer:
In $\triangle A B D$ and $\triangle B A C$,
$A D=B C$ (Given)
$\angle \mathrm{DAB}=\angle \mathrm{CBA}$ (Given)
$A B=B A$ (Common)
$\therefore \triangle A B D \cong \triangle B A C$ (By SAS congruence rule)
$\therefore B D=A C$ (By CPCT)
And, $\angle \mathrm{ABD}=\angle \mathrm{BAC}(\mathrm{By} \mathrm{CPCT})$
Question 3:
$A D$ and $B C$ are equal perpendiculars to a line segment $A B$ (See the given figure). Show that CD bisects $A B$.


Answer:
In $\triangle B O C$ and $\triangle A O D$,
$\angle \mathrm{BOC}=\angle \mathrm{AOD}$ (Vertically opposite angles)
$\angle C B O=\angle D A O\left(\right.$ Each $\left.90^{\circ}\right)$
$B C=A D$ (Given)
$\therefore \triangle \mathrm{BOC} \cong \triangle \mathrm{AOD}$ (AAS congruence rule)
$\therefore \mathrm{BO}=\mathrm{AO}$ (By CPCT)
$\Rightarrow C D$ bisects $A B$.

## Question 4:

$I$ and $m$ are two parallel lines intersected by another pair of parallel lines $p$ and $q$ (see the given figure). Show that $\triangle \mathrm{ABC} \cong \triangle C D A$.


Answer:
In $\triangle A B C$ and $\triangle C D A$,
$\angle \mathrm{BAC}=\angle \mathrm{DCA}$ (Alternate interior angles, as $p \| q$ )
$A C=C A(C o m m o n)$
$\angle B C A=\angle D A C$ (Alternate interior angles, as $/ \| m$ )
$\square \triangle \mathrm{ABC} \square \triangle \mathrm{CDA}$ (By ASA congruence rule)

## Question 5:

Line $l$ is the bisector of an angle $\square A$ and $B$ is any point on $l . B P$ and $B Q$ are perpendiculars from $B$ to the arms of $\square A$ (see the given figure). Show that:
(i) $\triangle \mathrm{APB} \square \triangle \mathrm{AQB}$
(ii) $\mathrm{BP}=\mathrm{BQ}$ or B is equidistant from the arms of $\square \mathrm{A}$.


Answer:
In $\triangle A P B$ and $\triangle A Q B$,
$\square A P B=\square A Q B$ (Each $90^{\circ}$ )
$\square \mathrm{PAB}=\square \mathrm{QAB}$ ( $/$ is the angle bisector of $\square \mathrm{A}$ )
$A B=A B$ (Common)
$\square \triangle \mathrm{APB} \square \triangle \mathrm{AQB}$ (By AAS congruence rule)
$\square B P=B Q(B y C P C T)$
Or, it can be said that $B$ is equidistant from the arms of $\square A$.

## Question 6:

In the given figure, $A C=A E, A B=A D$ and $\square B A D=\square E A C$. Show that $B C=D E$.


Answer:
It is given that $\square B A D=\square E A C$
$\square B A D+\square D A C=\square E A C+\square D A C$
$\square B A C=\square D A E$
In $\triangle B A C$ and $\triangle D A E$,
$A B=A D$ (Given)
$\square B A C=\square D A E$ (Proved above)
$A C=A E$ (Given)$\triangle \mathrm{BAC} \square \triangle \mathrm{DAE}$ (By SAS congruence rule)$B C=D E(B y C P C T)$

## Question 7:

$A B$ is a line segment and $P$ is its mid-point. $D$ and $E$ are points on the same side of $A B$ such that $\square B A D=\square A B E$ and $\square E P A=\square D P B$ (See the given figure). Show that (i) $\triangle \mathrm{DAP} \square \triangle \mathrm{EBP}$
(ii) $A D=B E$


Answer:
It is given that $\square E P A=\square D P B$
$\square \square E P A+\square D P E=\square D P B+\square D P E$
$\square \square \mathrm{DPA}=\square \mathrm{EPB}$
In $\triangle \mathrm{DAP}$ and $\triangle \mathrm{EBP}$,
$\square D A P=\square E B P$ (Given)
$A P=B P(P$ is mid-point of $A B)$
$\square D P A=\square E P B$ (From above)
$\square \triangle \mathrm{DAP} \square \triangle \mathrm{EBP}$ (ASA congruence rule)
$\square \mathrm{AD}=\mathrm{BE}(\mathrm{By} \mathrm{CPCT})$
Question 8:
In right triangle $A B C$, right angled at $C, M$ is the mid-point of hypotenuse $A B . C$ is joined to $M$ and produced to a point $D$ such that $D M=C M$. Point $D$ is joined to point $B$ (see the given figure). Show that:
(i) $\triangle \mathrm{AMC} \square \triangle \mathrm{BMD}$
(ii) $\square D B C$ is a right angle.
(iii) $\triangle \mathrm{DBC} \square \triangle \mathrm{ACB}$
(iv) $\mathrm{CM}=\frac{1}{2} \mathrm{AB}$


Answer:
(i) In $\triangle A M C$ and $\triangle B M D$,
$A M=B M$ ( $M$ is the mid-point of $A B$ )
$\square A M C=\square B M D$ (Vertically opposite angles)
CM = DM (Given)
$\square \triangle \mathrm{AMC} \square$ $\triangle \mathrm{BMD}$ (By SAS congruence rule)
$\square A C=B D(B y C P C T)$
And, $\square A C M=\square B D M$ (By CPCT)
(ii) $\square A C M=\square B D M$

However, $\square \mathrm{ACM}$ and $\square \mathrm{BDM}$ are alternate interior angles.
Since alternate angles are equal,
It can be said that DB || AC$\square D B C+\square A C B=180^{\circ}$ (Co-interior angles)$\triangle D B C+90^{\circ}=180^{\circ}$DBC $=90^{\circ}$
(iii) In $\triangle D B C$ and $\triangle A C B$,
$\mathrm{DB}=\mathrm{AC}$ (Already proved)
$\square D B C=\square A C B\left(\right.$ Each $\left.90^{\circ}\right)$
$B C=C B$ (Common)
$\square \triangle \mathrm{DBC} \square \triangle \mathrm{ACB}$ (SAS congruence rule)
(iv) $\triangle D B C \square \triangle A C B$
$\square A B=D C(B y C P C T)$
$\square A B=2 C M$
$\square \mathrm{CM}=\frac{1}{2} \mathrm{AB}$

## Exercise 7.2

## Question 1:

In an isosceles triangle $A B C$, with $A B=A C$, the bisectors of $\square B$ and $\square C$ intersect each other at O. Join $A$ to $O$. Show that:
(i) $O B=O C$ (ii) $A O$ bisects $\square A$

Answer:

(i) It is given that in triangle $A B C, A B=A C$
$\square \square \mathrm{ACB}=\square \mathrm{ABC}$ (Angles opposite to equal sides of a triangle are equal)
$\square^{\frac{1}{2}} \square A C B=\frac{\frac{1}{2}}{\square A B C}$$\square O C B=\square O B C$
$\square \mathrm{OB}=\mathrm{OC}$ (Sides opposite to equal angles of a triangle are also equal)
(ii) In $\triangle O A B$ and $\triangle O A C$,
$A O=A O$ (Common)
$A B=A C$ (Given)
OB = OC (Proved above)
Therefore, $\triangle \mathrm{OAB} \square \triangle \mathrm{OAC}$ (By SSS congruence rule)
$\square \square \mathrm{BAO}=\square \mathrm{CAO}$ (CPCT)
$\square$ AO bisects $\square$ A.
Question 2:
In $\triangle A B C, A D$ is the perpendicular bisector of $B C$ (see the given figure). Show that $\triangle A B C$ is an isosceles triangle in which $A B=A C$.


Answer:
In $\triangle A D C$ and $\triangle A D B$,
$A D=A D$ (Common)
$\square \mathrm{ADC}=\square \mathrm{ADB}$ (Each $90^{\circ}$ )
$C D=B D(A D$ is the perpendicular bisector of $B C)$
$\square \triangle \mathrm{ADC} \square \triangle \mathrm{ADB}$ (By SAS congruence rule)
$\square \mathrm{AB}=\mathrm{AC}$ (By CPCT)
Therefore, $A B C$ is an isosceles triangle in which $A B=A C$.

## Question 3:

$A B C$ is an isosceles triangle in which altitudes $B E$ and $C F$ are drawn to equal sides $A C$ and $A B$ respectively (see the given figure). Show that these altitudes are equal.


Answer:
In $\triangle A E B$ and $\triangle A F C$,
$\square$ AEB and $\square$ AFC (Each 90$)$
$\square \mathrm{A}=\square \mathrm{A}$ (Common angle)
$A B=A C$ (Given)
$\square \triangle \mathrm{AEB} \square \triangle \mathrm{AFC}$ (By AAS congruence rule)
$\square \mathrm{BE}=\mathrm{CF}$ (By CPCT)
Question 4:
$A B C$ is a triangle in which altitudes $B E$ and $C F$ to sides $A C$ and $A B$ are equal (see the given figure). Show that
(i) $\triangle \mathrm{ABE} \square \triangle \mathrm{ACF}$
(ii) $A B=A C$, i.e., $A B C$ is an isosceles triangle.


Answer:
(i) In $\triangle A B E$ and $\triangle A C F$,
$\square A B E$ and $\square A C F$ (Each $90^{\circ}$ )
$\square \mathrm{A}=$ $\qquad$ A (Common angle)
$B E=C F$ (Given)$\triangle A B E$$\triangle A C F$ (By AAS congruence rule)
(ii) It has already been proved that
$\triangle A B E \square \triangle A C F$$A B=A C(B y C P C T)$

## Question 5:

$A B C$ and $D B C$ are two isosceles triangles on the same base $B C$ (see the given figure). Show that $\square A B D=\square A C D$.


Answer:


Let us join $A D$.
In $\triangle A B D$ and $\triangle A C D$,
$A B=A C$ (Given)
$B D=C D$ (Given)
$A D=A D$ (Common side)
$\square \triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$ (By SSS congruence rule)
$\square \square \mathrm{ABD}=\square \mathrm{ACD}$ (By CPCT)

## Question 6:

$\triangle A B C$ is an isosceles triangle in which $A B=A C$. Side $B A$ is produced to $D$ such that $A D=A B$ (see the given figure). Show that $\square B C D$ is a right angle.

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Answer:
In $\triangle A B C$,
$A B=A C$ (Given)$\square A C B=\square A B C$ (Angles opposite to equal sides of a triangle are also equal)
In $\triangle A C D$,
$A C=A D$
$\square \square \mathrm{ADC}=\square \mathrm{ACD}$ (Angles opposite to equal sides of a triangle are also equal)
In $\triangle B C D$,
$\square \mathrm{ABC}+\square \mathrm{BCD}+\square \mathrm{ADC}=180^{\circ}$ (Angle sum property of a triangle)$\square A C B+\square A C B+\square A C D+\square A C D=180^{\circ}$
$\square 2(\square \mathrm{ACB}+\square \mathrm{ACD})=180^{\circ}$
$\square 2(\square B C D)=180^{\circ}$
$\square \square B C D=90^{\circ}$
Question 7:
$A B C$ is a right angled triangle in which $\square A=90^{\circ}$ and $A B=A C$. Find $\square B$ and $\square C$.
Answer:


It is given that
$A B=A C$
$\square \square C=\square B$ (Angles opposite to equal sides are also equal)
In $\triangle A B C$,
$\square A+\square B+\square C=180^{\circ}$ (Angle sum property of a triangle)
$\square 90^{\circ}+\square B+\square C=180^{\circ}$$90^{\circ}+\square B+\square B=180^{\circ}$$2 \square B=90^{\circ}$$\square B=45^{\circ}$$\square B=\square C=45^{\circ}$

## Question 8:

Show that the angles of an equilateral triangle are $60^{\circ}$ each.
Answer:


Let us consider that $A B C$ is an equilateral triangle.
Therefore, $A B=B C=A C$
$A B=A C$$\square C=\square B$
$B$ (Angles opposite to equal sides of a triangle are equal)
Also,
$A C=B C$$\square B=\square A$ $\square$ A (Angles opposite to equal sides of a triangle are equal)
Therefore, we obtain
$\square A=\square B=\square C$
In $\triangle A B C$,
$\square \mathrm{A}+\square \mathrm{B}+\square \mathrm{C}=180^{\circ}$
$\square \square \mathrm{A}+\square \mathrm{A}+\square \mathrm{A}=180^{\circ}$
$\square 3 \square \mathrm{~A}=180^{\circ}$$\square A=60^{\circ}$$\square A=$ $\square \mathrm{B}=\square$ $\square C=60^{\circ}$

Hence, in an equilateral triangle, all interior angles are of measure $60^{\circ}$.

## Exercise 7.3

## Question 1:

$\triangle A B C$ and $\triangle D B C$ are two isosceles triangles on the same base $B C$ and vertices $A$ and $D$ are on the same side of $B C$ (see the given figure). If AD is extended to intersect $B C$ at $P$, show that
(i) $\triangle A B D$$\triangle A C D$
(ii) $\triangle A B P$$\triangle A C P$
(iii) AP bisects $\square A$ as well as
(iv) $A P$ is the perpendicular bisector of $B C$.


Answer:
(i) In $\triangle A B D$ and $\triangle A C D$,
$A B=A C$ (Given)
$B D=C D$ (Given)
$A D=A D$ (Common)$\triangle A B D$$\triangle A C D$ (By SSS congruence rule)
$\square \square B A D=\square C A D$ (By CPCT)
$\square \square B A P=\square C A P$.... (1)
(ii) In $\triangle A B P$ and $\triangle A C P$,

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AB = AC (Given)
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$\square \mathrm{BAP}=\square \mathrm{CAP}$ [From equation (1)]
$A P=A P$ (Common)
$\square \triangle \mathrm{ABP} \square \triangle \mathrm{ACP}$ (By SAS congruence rule)
$\square \mathrm{BP}=\mathrm{CP}$ (By CPCT) ... (2)
(iii) From equation (1),
$\square \mathrm{BAP}=\square \mathrm{CAP}$
Hence, AP bisects $\square \mathrm{A}$.
In $\triangle B D P$ and $\triangle C D P$,
$B D=C D$ (Given)
DP = DP (Common)
$B P=C P[F r o m$ equation (2)]
$\square \triangle \mathrm{BDP} \square \triangle \mathrm{CDP}$ (By S.S.S. Congruence rule)
$\square \square \mathrm{BDP}=\square \mathrm{CDP}$ (By CPCT) ... (3)
Hence, AP bisects $\square \mathrm{D}$.
(iv) $\triangle \mathrm{BDP} \square \triangle \mathrm{CDP}$
$\square \square B P D=\square C P D$ (By CPCT) .... (4)
$\square \mathrm{BPD}+\square \mathrm{CPD}=180^{\circ}$ (Linear pair angles)
$\square \mathrm{BPD}+\square \mathrm{BPD}=180^{\circ}$
$2 \square \mathrm{BPD}=180^{\circ}$ [From equation (4)]
$\square \mathrm{BPD}=90^{\circ}$
From equations (2) and (5), it can be said that AP is the perpendicular bisector of $B C$.

Question 2:
$A D$ is an altitude of an isosceles triangles $A B C$ in which $A B=A C$. Show that
(i) AD bisects $B C$ (ii) AD bisects $\square A$.

Answer:

(i) In $\triangle B A D$ and $\triangle C A D$,
$\square \mathrm{ADB}=\square \mathrm{ADC}$ (Each $90^{\circ}$ as AD is an altitude)
$A B=A C$ (Given)
$A D=A D$ (Common)
$\square \triangle B A D \square \triangle C A D$ (By RHS Congruence rule)
$\square B D=C D$ (By CPCT)
Hence, AD bisects BC.
(ii) Also, by CPCT,
$\square B A D=\square C A D$
Hence, AD bisects $\square$ A.

## Question 3:

Two sides $A B$ and $B C$ and median $A M$ of one triangle $A B C$ are respectively equal to sides $P Q$ and $Q R$ and median $P N$ of $\triangle P Q R$ (see the given figure). Show that:
(i) $\triangle A B M \square \triangle P Q N$
(ii) $\triangle \mathrm{ABC} \square \triangle \mathrm{PQR}$


Answer:
(i) In $\triangle A B C, A M$ is the median to $B C$.

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$\square B M=\frac{1}{2} B C$
In $\triangle P Q R, P N$ is the median to $Q R$.
$\square \mathrm{QN}=\frac{1}{2} \mathrm{QR}$
However, $B C=Q R$
$\square \frac{1}{2} \mathrm{BC}=\frac{1}{2} \mathrm{QR}$
$\square \mathrm{BM}=\mathrm{QN}$..
In $\triangle A B M$ and $\triangle P Q N$,
$A B=P Q$ (Given)
$B M=Q N$ [From equation (1)]
$A M=P N$ (Given)$\triangle \mathrm{ABM} \square \triangle \mathrm{PQN}$ (SSS congruence rule)
$\square A B M=\square P Q N$ (By CPCT)
$\square A B C=\square P Q R . .$. (2)
(ii) In $\triangle A B C$ and $\triangle P Q R$,
$A B=P Q$ (Given)
$\square A B C=\square P Q R[$ From equation (2)]
$B C=Q R$ (Given)$\triangle A B C$$\Delta \mathrm{PQR}$ (By SAS congruence rule)

## Question 4:

$B E$ and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle $A B C$ is isosceles.

Answer:


In $\triangle B E C$ and $\triangle C F B$,
$\square \mathrm{BEC}=\square \mathrm{CFB}\left(\right.$ Each $90^{\circ}$ )
$B C=C B$ (Common)
$B E=C F$ (Given)
$\square \Delta \mathrm{BEC} \square \triangle \mathrm{CFB}$ (By RHS congruency)$\square B C E=\square C B F(B y C P C T)$$A B=A C$ (Sides opposite to equal angles of a triangle are equal)
Hence, $\triangle A B C$ is isosceles.

## Question 5:

$A B C$ is an isosceles triangle with $A B=A C$. Drawn $A P \square B C$ to show that $\square B=\square C$.
Answer:


In $\triangle A P B$ and $\triangle A P C$,
$\square \mathrm{APB}=\square \mathrm{APC}\left(\right.$ Each $90^{\circ}$ )
$A B=A C$ (Given)
$A P=A P$ (Common)$\square \mathrm{APB} \square \triangle \mathrm{A}$ $\triangle \mathrm{APC}$ (Using RHS congruence rule)
$\square \square \mathrm{B}=\square \mathrm{C}$ (By using CPCT)
Exercise 7.4

## Question 1:

Show that in a right angled triangle, the hypotenuse is the longest side.
Answer:


Let us consider a right-angled triangle $A B C$, right-angled at $B$.
In $\triangle A B C$,
$\square \mathrm{A}+\square \mathrm{B}+\square \mathrm{C}=180^{\circ}$ (Angle sum property of a triangle)$\square \mathrm{A}+90^{\circ}+\square \mathrm{C}=180^{\circ}$
$\square A+\square C=90^{\circ}$
Hence, the other two angles have to be acute (i.e., less than $90^{\circ}$ ).$\square B$ is the largest angle in $\triangle A B C$.$\square B>\square A$ andB > $\square$ C$A C>B C$ and $A C>A B$
[In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, $A C$ is the largest side in $\triangle A B C$.
However, $A C$ is the hypotenuse of $\triangle A B C$. Therefore, hypotenuse is the longest side in a right-angled triangle.

## Question 2:

In the given figure sides $A B$ and $A C$ of $\triangle A B C$ are extended to points $P$ and $Q$ respectively. Also, $\square \mathrm{PBC}<\square \mathrm{QCB}$. Show that $\mathrm{AC}>\mathrm{AB}$.


Answer:
In the given figure,

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\square \mathrm { ABC } + \square \mathrm { PBC } = 1 8 0 ^ { \circ } \text { (Linear pair)}
```\(\square \mathrm{ABC}=180^{\circ}-\square\) \(\square\) PBC ..

Also,
\(\square \mathrm{ACB}+\square \mathrm{QCB}=180^{\circ}\)
\(\square \mathrm{ACB}=180^{\circ}-\square \mathrm{QCB}\).
As \(\square \mathrm{PBC}<\square \mathrm{QCB}\),\(180^{\circ}-\square \mathrm{PBC}>180^{\circ}-\square \mathrm{QCB}\)
\(\square \square \mathrm{ABC}>\square \mathrm{ACB}\) [From equations (1) and (2)]
\(\square A C>A B\) (Side opposite to the larger angle is larger.)

\section*{Question 3:}

In the given figure, \(\square B<\square A\) and \(\square C<\square D\). Show that \(A D<B C\).


Answer:
In \(\triangle A O B\),
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\squareB<
\squareA

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\(\square A O<B O\) (Side opposite to smaller angle is smaller) ... (1)
In \(\triangle C O D\),
\(\square C<\square D\)
\(\square\) OD < OC (Side opposite to smaller angle is smaller) ...
On adding equations (1) and (2), we obtain
\(A O+O D<B O+O C\)
\(A D<B C\)
Question 4:
\(A B\) and \(C D\) are respectively the smallest and longest sides of a quadrilateral \(A B C D\) (see the given figure). Show that \(\square A>\square C\) and \(\square B>\square D\).


Answer:


Let us join AC.

In \(\triangle A B C\),
\(A B<B C\) (AB is the smallest side of quadrilateral \(A B C D)\)
\(\square \square 2<\square 1\) (Angle opposite to the smaller side is smaller) ..
In \(\triangle \mathrm{ADC}\),
\(A D<C D\) (CD is the largest side of quadrilateral \(A B C D\) )
\(\square \square 4<\square 3\) (Angle opposite to the smaller side is smaller) ...
On adding equations (1) and (2), we obtain
\(\square 2+\square 4<\square 1+\square 3\)\(\square C<\square A\)\(\square A>\square C\)
Let us join BD.


In \(\triangle A B D\),
\(A B<A D\) ( \(A B\) is the smallest side of quadrilateral \(A B C D\) )
\(\square \square 8<\square 5\) (Angle opposite to the smaller side is smaller) ...
In \(\triangle B D C\),
\(B C<C D\) (CD is the largest side of quadrilateral \(A B C D\) )
\(\square \square 7<\square 6\) (Angle opposite to the smaller side is smaller) ...
On adding equations (3) and (4), we obtain
\(\square 8+\square 7<\square 5+\square 6\)\(\square \square<\square B\)
\(\square \square \mathrm{B}>\square \mathrm{D}\)

\section*{Question 5:}

In the given figure, \(P R>P Q\) and \(P S\) bisects \(\square Q P R\). Prove that \(\square P S R>\square P S Q\).


Answer:
As \(P R>P Q\),
\(\square \square \mathrm{PQR}>\square\) PRQ (Angle opposite to larger side is larger) ... (1)
PS is the bisector of \(\square \mathrm{QPR}\).\(\square\) QPS = \(\square\) RPS ..
\(\square P S R\) is the exterior angle of \(\triangle \mathrm{PQS}\).\(\square P S R=\square P Q R+\square Q P S\)
\(\square P S Q\) is the exterior angle of \(\triangle P R S\).\(\square P S Q=\square P R Q+\square R P S\).
Adding equations (1) and (2), we obtain
\(\square \mathrm{PQR}+\square \mathrm{QPS}>\square \mathrm{PRQ}+\square \mathrm{RPS}\)
\(\square \square \mathrm{PSR}>\square \mathrm{PSQ}\) [Using the values of equations (3) and (4)]

\section*{Question 6:}

Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Answer:


Let us take a line \(I\) and from point \(P\) (i.e., not on line \(I\) ), draw two line segments PN and PM. Let PN be perpendicular to line \(I\) and \(P M\) is drawn at some other angle.

In \(\triangle P N M\),
\(\square \mathrm{N}=90^{\circ}\)\(\square \mathrm{P}+\square \mathrm{N}+\)\(M=180^{\circ}\) (Angle sum property of a triangle)\(\square \mathrm{P}+\square \mathrm{M}=90^{\circ}\)
Clearly, \(\square \mathrm{M}\) is an acute angle.\(\square \mathrm{M}<\square \mathrm{N}\)PN < PM (Side opposite to the smaller angle is smaller)
Similarly, by drawing different line segments from \(P\) to \(l\), it can be proved that \(P N\) is smaller in comparison to them.
Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

\section*{Exercise 7.5}

\section*{Question 1:}
\(A B C\) is a triangle. Locate a point in the interior of \(\triangle A B C\) which is equidistant from all the vertices of \(\triangle A B C\).

Answer:
Circumcentre of a triangle is always equidistant from all the vertices of that triangle. Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.


In \(\triangle A B C\), we can find the circumcentre by drawing the perpendicular bisectors of sides \(A B, B C\), and \(C A\) of this triangle. \(O\) is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of \(\triangle A B C\).

\section*{Question 2:}

In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Answer:
The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.

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Here, in \(\triangle A B C\), we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of \(\triangle A B C\).

\section*{Question 3:}

In a huge park people are concentrated at three points (see the given figure)

\(\mathrm{B}^{\bullet}\)
\[
{ }^{\circ} \mathrm{C}
\]

A: where there are different slides and swings for children,
B: near which a man-made lake is situated,
C: which is near to a large parking and exit.
Where should an ice-cream parlour be set up so that maximum number of persons can approach it?
(Hint: The parlor should be equidistant from \(A, B\) and \(C\) )
Answer:
Maximum number of persons can approach the ice-cream parlour if it is equidistant from \(A, B\) and \(C\). Now, \(A, B\) and \(C\) form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre \(O\) of \(\triangle A B C\).

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In this situation, maximum number of persons can approach it. We can find circumcentre \(O\) of this triangle by drawing perpendicular bisectors of the sides of this triangle.

\section*{Question 4:}

Complete the hexagonal and star shaped rangolies (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?


Answer:
It can be observed that hexagonal-shaped rangoli has 6 equilateral triangles in it.

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Area of \(\triangle \mathrm{OAB}=\frac{\sqrt{3}}{4}(\text { side })^{2}=\frac{\sqrt{3}}{4}(5)^{2}\)
\(=\frac{\sqrt{3}}{4}(25)=\frac{25 \sqrt{3}}{4} \mathrm{~cm}^{2}\)
Area of hexagonal-shaped rangoli \(=6 \times \frac{25 \sqrt{3}}{4}=\frac{75 \sqrt{3}}{2} \mathrm{~cm}^{2}\)
Area of equilateral triangle having its side as \(1 \mathrm{~cm}=\frac{\sqrt{3}}{4}(1)^{2}=\frac{\sqrt{3}}{4} \mathrm{~cm}^{2}\)
Number of equilateral triangles of 1 cm side that can be filled
in this hexagonal-shaped rangoli \(=\frac{\frac{75 \sqrt{3}}{2}}{\frac{\sqrt{3}}{4}}=150\)
Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.


Area of star-shaped rangoli \(=12 \times \frac{\sqrt{3}}{4} \times(5)^{2}=75 \sqrt{3}\)

Number of equilateral triangles of 1 cm side that can be filled
in this star-shaped rangoli \(=\frac{75 \sqrt{3}}{\frac{\sqrt{3}}{4}}=300\)
Therefore, star-shaped rangoli has more equilateral triangles in it.```

